**Home Assignment 1**

Ilana Pervoi, Pan Eyal

1. **Minimizations with different norms lead to different answer:**

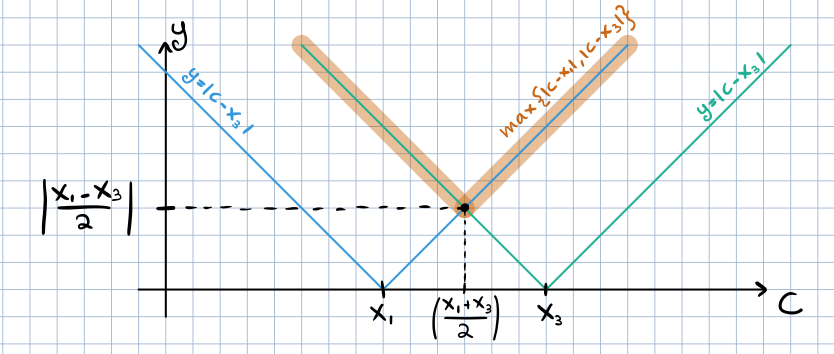
We will mark

By setting it to be 0, we will find the critical point (which is a minimum) as required:

Because we can conclude that:

we will prove that:

:



1. If then:

So:

1. If then:
2. If then:

So:

Therefore: will set the minimal value for

we will prove that :

1. If then

So:

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1. **Least Squares:**
2. We compute the code in python using NumPy package.

By calculating the solution x by the normal equation:

we receive

1. No, we won’t receive a unique solution.

is singular, hence as we’ve learned we will receive infinitely many solutions.

Furthermore, we can see that for each ( from the singularity properties), ,when x is the solution we already found, is also a valid LS solution.

proof:

(And because :)

To find the minimal loss, we compute the code by calculating the following r:

Then, by calculating its norm we determine its size:

1. For the 1st equation to almost get exactly satisfied (when ),

we’ve made a loop that will change the weight of the 1st equation by adding to the LS equation a “weighted matrix” we called that has bigger weight at index .

Meaning, is in the form of:

To find the minimal value to satisfy the equation, and that , we’ve made a while loop that increases value by with each iteration, and checks for loss.

We’ve exited the loop when the loss was less then .

We found that it occurred when

1. **Eigenvalues and positive definite matrices:**
2. Let be the eigenvalue of matric .

By definition,

Which means is an eigenvalue of the matrix as required.

1. Let .

**Support claim**:

Proof:

Let

Let

By norm properties

And so

**Main claim**:

Assume is a full rank.

By support claim

Assume is invertible.

By support claim

1. We saw in the previous question that .

By proving that and transitivity properties, we will conclude that .

Firstly, because its transpose is also :

Assume is invertible.

To show that we need to prove that

From norm properties

Finally, because and so

Hence

Assume is positive definite.

By definition,

Let , then

1. Assume .

We need to prove that is always positive definite.

Once again, we’ll do so by showing that .

Let .

We will notice that

And by norm properties

Also,

therefore:

In conclusion,

1. **Frobenius Norm:**
2. We need to prove that

**Proof**:

By matrix multiplication definition: :

For

For: ,

Therefore:

1. We want to find such that

And from the previous proof

Hence

We will perform derivative and find minimal value:

In other words,

The solution X for the equation will set the minimal value for .

The solution will be unique when is invertible.

1. To find the solution for the matrices we will find, for each row of and , a LS solution for the equation : .

By calculating, we found out that

1. **Working with real Data:**

At the beginning, we choose all the following countries to work with:

Belgium, Germany, Portugal, Spain, Switzerland, Italy and United Kingdom,

We choose to work with 80% of the data to create our model, by taking the first 189 days.

We defined to be the matrix that represent the 1st 80% of those countries data in the following way:

* each column of the matrix will represent a country.
* each row will represent the new cases of covid-19 divided by the density of that country.

We defined to be a vector that represents France’s data similarly.

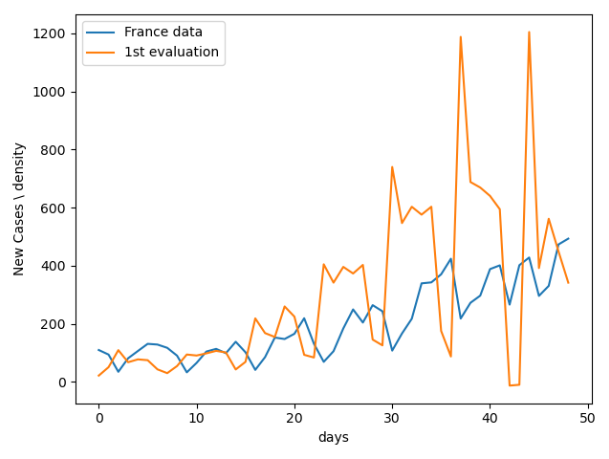
Then, we calculated the LS solution and for the equation:

We got that with a minimal error of:

We defined to be the remaining 20% and then applied our model on it.

So:

We received the following graph:

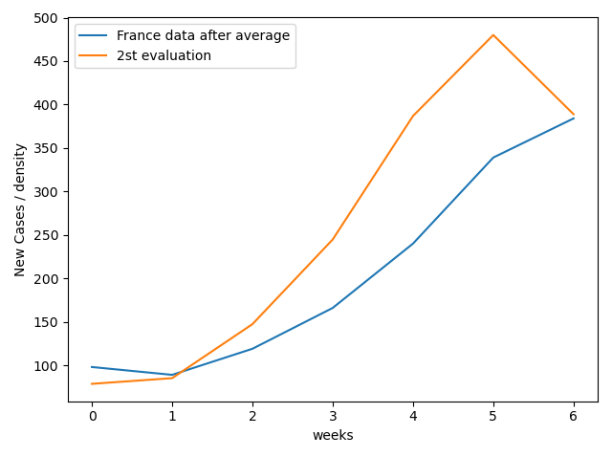


We can see that the graph has a lot of noise, therefore we chose to average it by weeks and normalize it by density of each country.

By repeating the process explained above, with the new reformed data

We got that with a minimal error of:

We received the following graph:

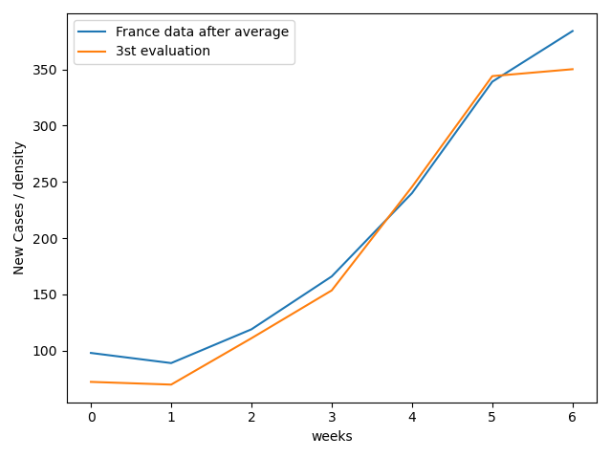


At this point, we noticed that the graph was not as accurate as we hoped it would be.

By looking at the countries data we saw that Belgium had the highest population density compared to France and decided to remove it. Then, we repeated the process once again.

We received that with a minimal error of:

We received the following graph which is quite close to the original data:



At the end, we calculated the ratio between our model error and the real data and received that the ratio is: **0.00087826**.